University of Toledo, Department of Physics and Astronomy

# Ph.D. Qualifying Exam

Fall 2019
September 28

Instructions:

## CLASSICAL MECHANICS (CM):

1. A rod of mass *m* and length *l* is suspended from one end. The other end is attached to a horizontal massless spring with spring constant *k*. Using the small-angle approximation determine the oscillation frequency of the rod.

2. A particle of mass *m* hangs from a spring with spring constant *k*. A second particle of mass *m* is suspended from the first particle by a spring which also has spring constant *k*. Ignoring gravity (since this only shifts the point of equilibrium) determine the normal mode frequencies.

3. A projectile of mass *m* is launched towards the North pole with speed *v* at an angle from the horizontal from a point in the northern Hemisphere with latitude . Assuming that the angular velocity of the Earth is given by , and that the trajectory is short enough that the Earth can be taken to be flat and neglecting air resistance, determine an expression for the Eastward deflection of the projectile when it lands. Hint: for simplicity neglect the contribution to the Coriolis force fro .

## <u>ELECTRICITY AND MAGNETISM (E&M):</u>

**1**. Imagine the reflection of a light wave incident onto a sheet of glass, where the x-z plane defines the surface of the glass and the polarization vector of the wave is

**2**. Optical tweezers can be used to manipulate microscopic objects using the intense electric field created by a focused light beam. In this problem, you will show that the force on a small dielectric particle in the electric and magnetic fields of the light beam is given by:

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a. determine the net force on a dipole with a polarization 2&Λ M∉&Λ MT&Φ T&, assuming an electric field ϶& and magnetic field ∃&. Also assume that the size of the particle is much smaller than the wavelength of light. To do this, write the net Lorentz force as a function of 2&, ϶& and ∃&, and then use the equation 2&Λ ∧϶& to write it in terms of ∧( ∧ Λ (4 | ) , ϶& and ∃&.

b. use the equality

κ϶& ∉&϶&Λ ∉&λ σ/τ ϶ 6 π Φ ϶&Η κ∉&Η ϶&

for the curl of ϶& to write the force in terms of ∉&϶ 6 and the time derivative of the Poynting vector. Since the flux does not change, the time derivative of the Poynting term is zero.

c. demonstrate that the dielectric object will be pushed to the peak of the electric field in the center of the tweezers.

**3**. Imagine a cosmic ray moving through a magnetic field in an interstellar cloud.

a. The kinetic energy of the cosmic ray, which is a proton (mass = $1.67 \times 10^{27}$ kg), is $10^7$ eV (1 eV = $1.6 \times 10^{-12}$ erg). What is the velocity of the particle? (Ignore relativistic effects.)

b. The magnetic field is 30 $\mu$G, where $10^4$ Gauss = 1 Tesla. Given that the charge of the electron is $1.6 \times 10^{19}$ Coulombs, calculate the cyclotron frequency of the proton.

c. Assuming that the direction of the particle is rotated by an angle   relative to the magnetic field vector, what is the radius of the cyclotron orbits?

d. Write the equation for the position of the particle as it spirals through the cloud. Adopt a starting point of

## QUANTUM MECHANICS (QM):

1. Based on QM, explain why the electron binding energy in He atom is greater than that in H and Li atoms? In the order of magnitude, what is the difference in these binding energies?

2. The wave function

   $$= C[2e^{5i} \quad - e^{-2i} \quad ]$$

   describes a plane rotator (  is the angle of rotation). An experiment is conducted to determine the angular momentum *L* of the rotator.
   a) What possible values can one find for the measured angular momentum?
   b) What are the probabilities of finding these values?
   c) Determine the value of constant *C*.

3. Consider a system that is initially in the state

   $$( \; , \; ) = A[Y_{1,-1}( \; , \; ) + 2Y_{10}( \; , \; ) + 3Y_{11}( \; , \; )]$$

   where *A* is a normalization constant.
   (a) Find